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FUZZY SET: A MEASURE OF IMPRECISION

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ABSTRACT

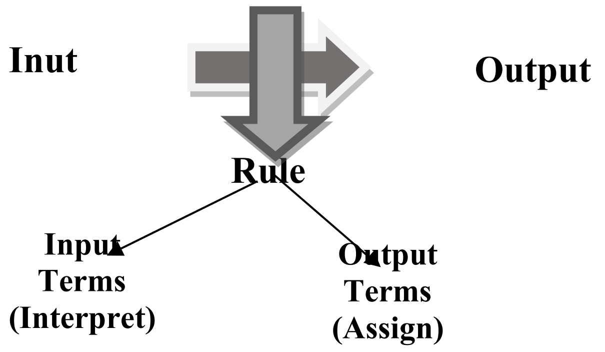
Classical logic only permits conclusions which are either true or false. However, there are also propositions with variable answers. In such instances, the truth appears as the result of reasoning from inexact or partial knowledge in which the sampled answers are mapped on a spectrum. Humans and animals often operate using fuzzy evaluations in many everyday situations. Both degrees of truth and probabilities range between 0 and 1 and hence may seem similar at first, but fuzzy logic uses degrees of truth as a mathematical model of vagueness, while probability is a mathematical model of uncertainty. Fuzzification operations can map mathematical input values into fuzzy membership functions. And the opposite de-fuzzifying operations can be used to map a fuzzy output membership functions into a "crisp" output value that can be then used for decision or control purposes.

Keywords- *Membership function, Fuzzy-fication, De-fuzzy-fication.*

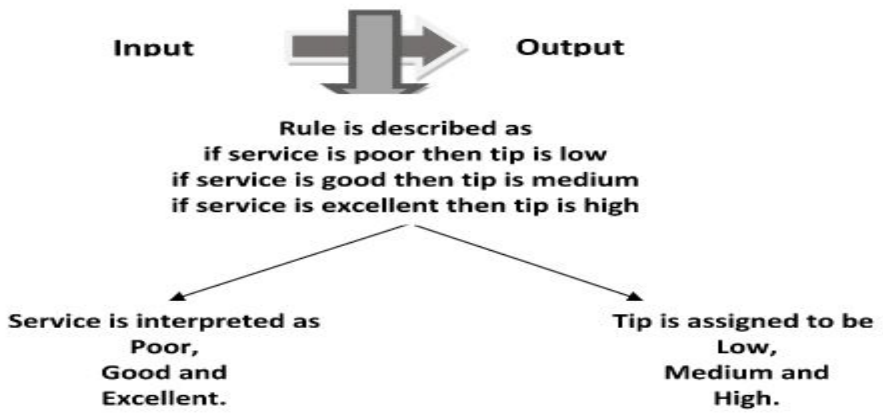
1. INTRODUCTION

In most of the realistic decision making problems, we come across with some vague/ imprecise terms which play an important role in human thinking. In this paper, we have tried to present the basic properties and implications of a concept which would be used in dealing with such imprecise terms. It requires a natural way of dealing with problems in which the source of imprecision is the absence of sharply defined criteria.

Let us begin the discussion with a general and specified description of an imprecise decision (inference) system. The following diagram depicts a road map for the decision process. Usually the flowchart of general decision making model can be given as below;



Based on this model a specific example is considered to highlight the rational decision process



So, the decision process can be interpreted as a method to map an input space to an output space and the primary mechanism for doing this is a list of if – then statements called rules. The rules themselves are useful because they refer to variables and the adjectives that describe these variables. All rules are evaluated in parallel. However the order of the rules is unimportant. Here, the input as well as the output variables and its adjectives cannot be defined precisely. Such a situation is very elegantly tackled with the use of fuzzy logic and fuzzy logic begins with the concept of a fuzzy set.

2. WHATS IS A FUZZY SET?

Before going into detail it would be better to review the concepts of crisp set and fuzzy set.

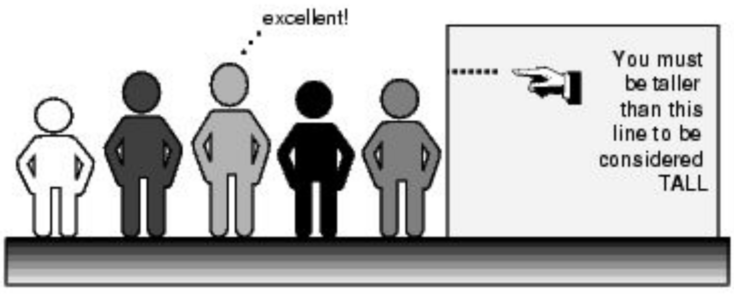
2.1 Crisp sets vs. fuzzy sets

In a crisp set, an element is either a member of the set or not. For example, Ramesh belongs in the class of boys but Nisha does not.

Fuzzy sets, on the other hand, allow elements to be *partially* in a set. Each element is given a degree of membership in a set. This membership value can range from 0 (not an element of the set) to 1 (a member of the set).

It is clear that if one only allowed the extreme membership values of 0 and 1, that this would actually be equivalent to crisp sets. A membership function is the relationship between the values of an element and its degree of membership in a set.

One of the most commonly used examples of a fuzzy set is the set of tall people. In this case the universe of discourse is all potential heights, say from 3 feet to 9 feet, and the word "tall" would correspond to a curve that defines the degree to which any person is tall. If the set of tall people is given the well-defined (crisp) boundary of a classical set, we might say all people taller than six feet are officially considered tall. But such a distinction is clearly absurd. It may make sense to consider the set of all real numbers greater than six because numbers belong on an abstract plane, but when we want to talk about real people, it is unreasonable to call one person short and another one tall when they differ in height by the width of a hair.



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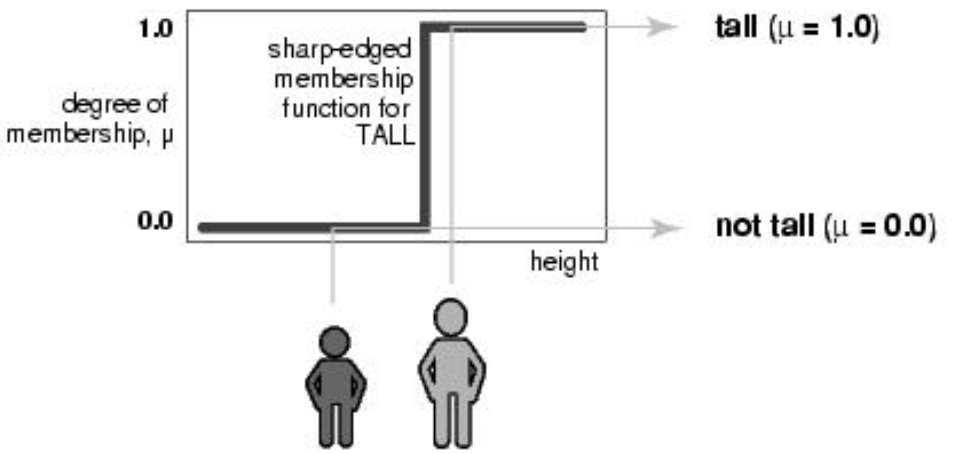
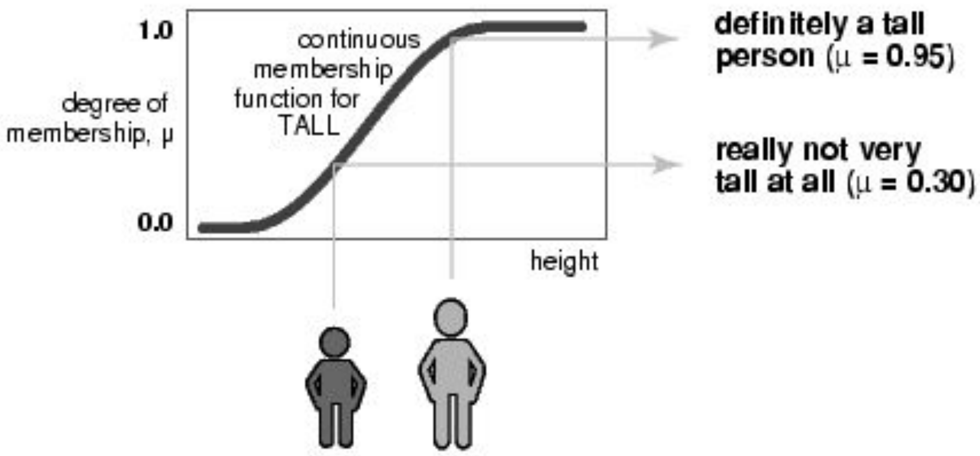
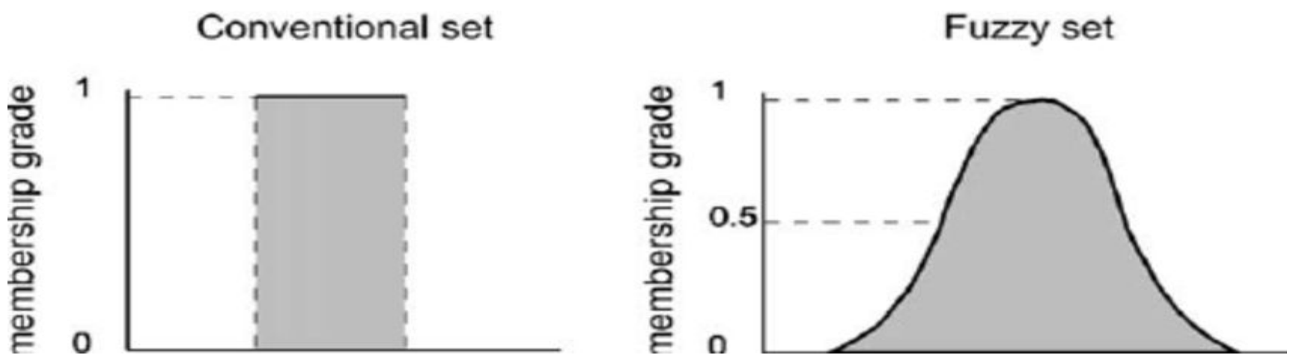
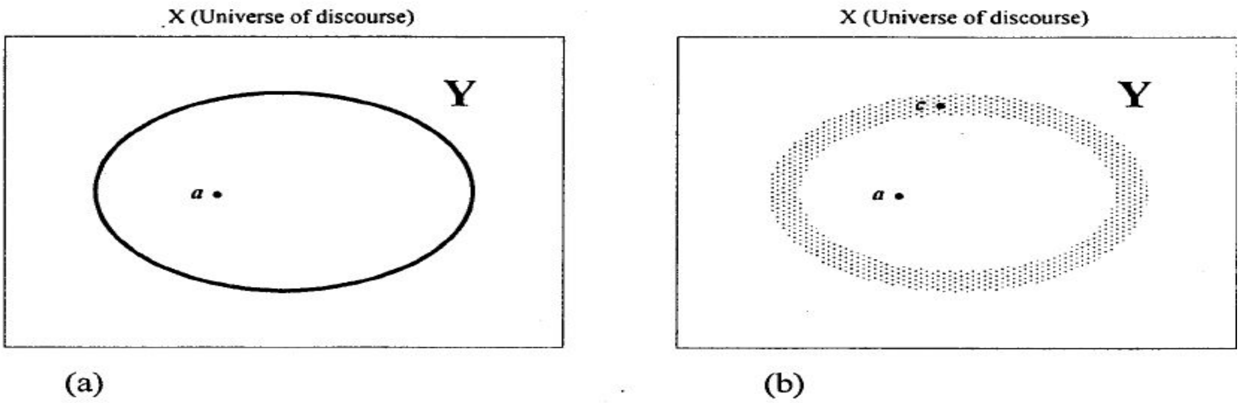




The kind of distinction shown above is unworkable, then what is the right way to define the set of tall people? The above figure shows a smoothly varying curve that passes from not-tall to tall. The output-axis is a number known as the membership value between 0 and 1. The curve is known as a membership function and is often given the designation of µ. This curve defines the transition from not tall to tall. Both people are tall to some degree, but one is significantly less tall than the other.

Subjective interpretations and appropriate units are built right into fuzzy sets. If I say "She's tall," the membership function "tall" should already take into account whether I'm referring to a six-year-old or a grown woman. Similarly, the units are included in the curve. Certainly it makes no sense to say "Is she tall in inches or in meters?" A comparison between conventional set and its equivalent fuzzy set can be seen below.



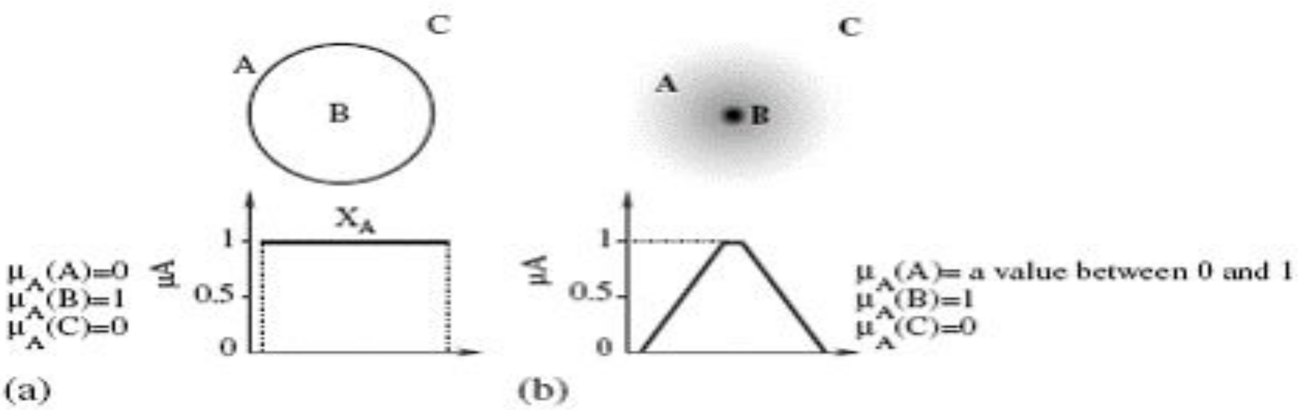


We may conclude that

i) in any crisp set A, each element x is an element of A (100%) or is not (0%), whereas

ii) in a fuzzy set A, each item x has a degree of membership in the set A. Also, the item can have nonzero membership in both A and ~A (complement).

This can be depicted pictorially as below:



2.2 Membership Functions

The only condition a membership function must really satisfy is that it must vary between 0 and 1. The function itself can be an arbitrary curve whose shape we can define as a function that suits us from the point of view of simplicity, convenience, speed, and efficiency.

A classical set might be expressed as

$A = \{x \mid x > 6\}$

A fuzzy set is an extension of a classical set. If X is the universe of discourse and its elements are denoted by *x*, then a fuzzy set A in X is defined as a set of ordered pairs.

$A = \{x, \mu_A(x) \mid x \in X\}$

$\mu_A(x)$ is called the membership function (or MF) of *x* in A. The membership function maps each element of X to a membership value between 0 and 1.

The simplest membership functions are formed using straight lines. Of these, the simplest is the *triangular* membership function, and it has the function name trimf. It's nothing more than a collection of three points forming a triangle.

The *trapezoidal* membership function, trapmf, has a flat top and really is just a truncated triangle curve. These straight line membership functions have the advantage of simplicity.

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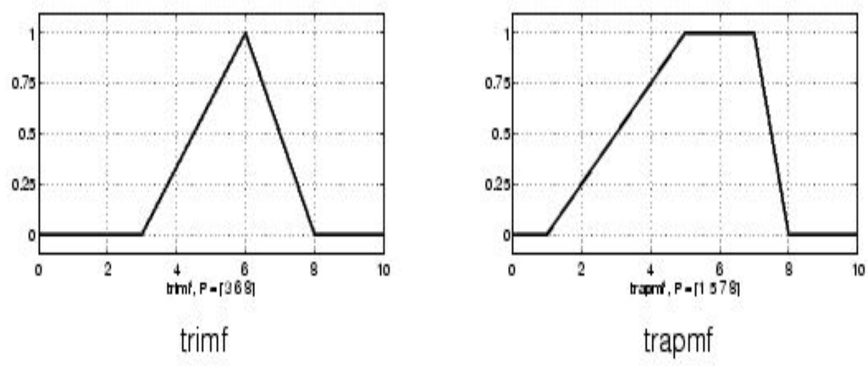
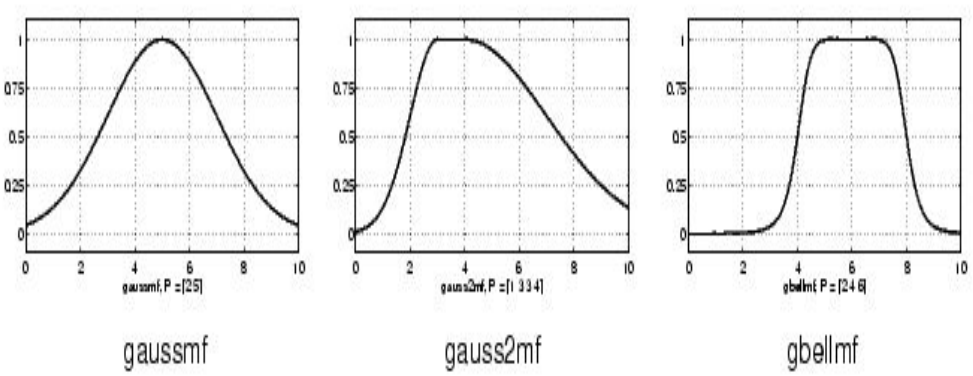


Two membership functions are built on the *Gaussian* distribution curve: a simple Gaussian curve and a two-sided composite of two different Gaussian curves. The two functions are gaussmf and gauss2mf.

The *generalized bell* membership function is specified by three parameters and has the function name gbellmf. The bell membership function has one more parameter than the Gaussian membership function, so it can approach a non-fuzzy set if the free parameter is tuned. Because of their smoothness and concise notation, Gaussian and bell membership functions are popular methods for specifying fuzzy sets. Both of these curves have the advantage of being smooth and nonzero at all points.



There's a very wide selection to choose from when you're selecting your favorite membership function.

2.3 **Summary of Membership Functions**

- Fuzzy sets describe vague concepts (tall girl, good service, low payment).
- A fuzzy set admits the possibility of partial membership in it. (150 cm is sort of a tall girl, 5% of the bill amount is rather less tip).
- The degree an object belongs to a fuzzy set is denoted by a membership value between 0 and 1. (150 cm hight to the degree 0.8).
- A membership function associated with a given fuzzy set maps an input value to its appropriate membership value.

3. Fuzzy Mathematical Operations

In making a fuzzy rule, we use the concept of "and", "or", and sometimes "not". The sections below describe the most common definitions of these "fuzzy combination" operators. Fuzzy combinations are also referred to as "T-norms".

3.1 Fuzzy "and"

The fuzzy "and" is written as:

$$\mu_{A\cap B} = T(\mu_A(x), \mu_B(x))$$

where μ_A is read as "the membership in class A" and μ_B is read as "the membership in class B". There are many ways to compute "and". The two most common are:

(i) Zadeh - min(μ_A(x) , μ_B(x))This technique, named after the inventor of fuzzy set theory simply computes the "and" by taking the minimum of the two (or more) membership values. This is the most common definition of the fuzzy "and".

(ii) Product - μ_A(x) times μ_B(x) This techniques computes the fuzzy "and" by multiplying the two membership values.

Both techniques have the following two properties:
One of the nice things about both definitions is that they also can be used to compute the Boolean "and". Table 1 shows the Boolean "and" operation. Notice that both fuzzy "and" definitions also work for these numbers. The fuzzy "and" is an extension of the Boolean "and" to numbers that are not just 0 or 1, but between 0 and 1.

Input1 (A)	Input2 (B)	Output (A "and" B)
0	0	0
0	1	0
1	0	0
1	1	1

Table 1: The Boolean "and"

3.2 Fuzzy "or"
The fuzzy "or" is written as:

$$\mu_{A \cup B} = T(\mu_A(x), \mu_B(x))$$

Similar to the fuzzy "and", there are two techniques for computing the fuzzy "or":
(i) Zadeh - max(μ_A(x) , μ_B(x)) This technique computes the fuzzy "or" by taking the maximum of the two (or more) membership values. This is the most common method of computing the fuzzy "or".
(ii) Product - (μ_A(x) + μ_B(x) - (μ_A(x) . μ_B(x)) This technique uses the difference between the sum of the two (or more) membership values and the product of the membership values.
Similar to the fuzzy "and", both definitions of the fuzzy "or" also can be used to compute the Boolean "or". Table 2 shows the Boolean "or" operation. Notice that both fuzzy "or" definitions also work for these numbers. The fuzzy "or" is an extension of the Boolean "or" to numbers that are not just 0 or 1, but between 0 and 1.

Input1 (A)	Input2 (B)	Output (A "or" B)
0	0	0
0	1	1
1	0	1
1	1	1

Table 2: The Boolean "or"

3.3 Fuzzy "not"
The fuzzy "not" is written as:

$$\mu_{\sim A}(x) = 1 - \mu_A(x)$$

Similar to both the fuzzy "and" & the fuzzy "or", fuzzy "not" can also be used to compute the Boolean "not" as shown in Table 3. Notice that fuzzy "not" definitions also work for these numbers. The fuzzy "not" is an extension of the Boolean "not" to numbers that are not just 0 or 1, but between 0 and 1.

Input (A)	Output (not A)
0	1
1	0

Table 3: The Boolean "not"

4. FUZZY INTERFERENCE SYSTEM

A fuzzy inference system (FIS) is a system that uses fuzzy set theory to map inputs (*features* in the case of fuzzy classification) to outputs (*classes* in the case of fuzzy classification). One of the FIS is discussed herewith.
To compute the output of a FIS given the inputs, one must go through six steps:
1. Determining a set of fuzzy rules
2. Fuzzifying the inputs using the input membership functions,
3. Combining the fuzzified inputs according to the fuzzy rules to establish a rule strength,
4. Finding the consequence of the rule by combining the rule strength and the output membership function,
5. Combining the consequences to get an output distribution, and
6. Defuzzifying the output distribution (this step is only if a crisp output (class) is needed).

4.1 Creating fuzzy rules

Fuzzy rules are a collection of linguistic statements that describe how the FIS should make a decision regarding classifying an input or controlling an output. Fuzzy rules are always written in the following form:
if *(input1 is membership function1)* ***and/or*** *(input2 is membership function2)* ***and/or*** ***then*** *(output$_n$ is output membership function$_n$).*
For example, one could make up a rule that says: ***if*** *temperature is high* ***and*** *humidity is high* ***then*** *room is hot.*
There would have to be membership functions that define what we mean by high temperature (input1), high humidity (input2) and a hot room (output1). This process of taking an input such as temperature and processing it through a membership function to determine what we mean by "high" temperature is called fuzzification. Also, we must define what we mean by "and" / "or" in the fuzzy rule. This is called fuzzy combination and is discussed in section.

4.2 Fuzzification

The purpose of fuzzification is to map the inputs to values from 0 to 1 using a set of input membership functions. These inputs are mapped into fuzzy numbers by drawing a line up from the inputs to the input membership functions above and marking the intersection point.
These input membership functions, as discussed previously, can represent fuzzy concepts such as "large" or "small", "old" or "young", "hot" or "cold", etc. When choosing the input membership functions, the definition of what we mean by "large" and "small" may be different for each input.

4.3 Defuzzification of Output Distribution

In many instances, it is desired to come up with a single crisp output from a FIS. For example, if one was trying to classify a letter drawn by hand on a drawing tablet, ultimately the FIS would have to come up with a crisp number to tell the computer which letter was drawn. This crisp number is obtained in a process known as defuzzification. There are many techniques for defuzzifying.

5. ORIGIN AND ETYMOLOGY

The intellectual origins of the idea of fuzzy concepts have been traced to a diversity of famous and well known thinkers including Plato, Georg Wilhelm Friedrich Hegel, Karl Marx, Friedrich Engels, Friedrich Nietzsche, Jan Lukasiewicz, Alfred Tarski, Stanisław Jaskowski and Donald Knuth. This suggests that the idea of fuzzy concepts has, in one form or another, a very long history in human thought.
However, usually the Iranian born, American computer scientist Lotfi A. Zadeh is credited with inventing the specific idea of a "fuzzy concept" in his seminal 1965 paper on fuzzy sets, because he gave a formal mathematical presentation of the phenomenon which was widely accepted by scholars. It was also Zadeh who played a decisive role in developing the field of fuzzy logic, fuzzy sets and fuzzy systems, with a large number of scholarly papers.
In fact, the German scholar Dieter Klaua [1] also published a German-language paper on fuzzy sets in 1965, but he used a different terminology (he referred to "many-valued sets"). An earlier attempt to create a theory of sets where set membership is a matter of degree was made by Abraham Kaplan and Hermann Schott in 1951. They intended to apply the idea to empirical research. Kaplan and Schott measured the degree of membership of empirical classes using real numbers between 0 and 1, and they defined corresponding notions of intersection, union, complementation and subset. However, at the time, their idea "fell on stony ground".
Radim Belohlavek explains it as "There exists strong evidence, established in the 1970s in the psychology of concepts... that human concepts have a graded structure in that whether or not a concept applies to a given object is a matter of degree, rather than a yes-or-no question, and that people are capable of working with the degrees in a consistent way. This finding is intuitively quite appealing, because people say "this product is more or less good" or "to a certain degree, he is a good athlete", implying the graded structure of concepts. In his classic paper, Zadeh [3] [4], Zimmermann [2] called the concepts with a graded structure fuzzy concepts and argued that these concepts are a rule rather than an

exception when it comes to how people communicate knowledge. Moreover, he argued that to model such concepts mathematically is important for the tasks of control, decision making, pattern recognition, and the like. Zadeh [5] proposed the notion of a fuzzy set that gave birth to the field of fuzzy logic.

6. APPLICATIONS

In philosophical logic, fuzzy concepts are often regarded as concepts which in their application, or formally speaking, are neither completely true nor completely false, or which are partly true and partly false; they are ideas which require further elaboration, specification or qualification to understand their applicability (the conditions under which they truly make sense).

In mathematics and statistics, a fuzzy variable (such as "the temperature", "hot" or "cold") is a value which could lie in a probable range defined by quantitative limits or parameters, and which can be usefully described with imprecise categories (such as "high", "medium" or "low") using some kind of qualitative scale.

In mathematics and computer science, the gradations of applicable meaning of a fuzzy concept are described in terms of quantitative relationships defined by logical operators. Such an approach is sometimes called "degree-theoretic semantics" by logicians and philosophers, but the more usual term is fuzzy logic or many-valued logic. The novelty of fuzzy logic is, that it "breaks with the traditional principle that formalisation should correct and avoid, but not compromise with, vagueness".

Fuzzy reasoning (i.e., reasoning with graded concepts) turns out to have many practical uses. It is nowadays widely used in the programming of vehicle and transport electronics, household appliances, video games, language filters, robotics, all kinds of control systems, and various kinds of electronic equipment used for pattern recognition, surveying and monitoring (such as radars).

Fuzzy reasoning is also used in artificial intelligence, virtual intelligence and soft computing research. "Fuzzy risk scores" are used by project managers and portfolio managers to express risk assessments. Fuzzy logic has even been applied to the problem of predicting cement strength. It looks like fuzzy logic will eventually be applied in almost every aspect of life, even if people are not aware of it, and in that sense fuzzy logic is an astonishingly successful invention.

A lot of research on fuzzy logic was done by Japanese researchers inventing new machinery, electronic equipment and appliances. Zimmermann [2] has contributed a lot in the field of programming. The North American Fuzzy Information Processing Society (NAFIPS) was founded in 1981. There also exists an International Fuzzy Systems Association (IFSA). In Europe, there is a European Society for Fuzzy Logic and Technology (EUSFLAT).

Lotfi Zadeh estimates there are more than 50,000 fuzzy logic-related, patented inventions. He lists 28 journals dealing with fuzzy reasoning, and 21 journal titles on soft computing. There are now close to 100,000 publications with the word "fuzzy" in their titles, or maybe even 300,000.

7. CONCLUSION

A fuzzy concept is a concept of which the boundaries of application can vary considerably according to context or conditions, instead of being fixed once and for all. This means the concept is vague in some way, lacking a fixed, precise meaning, without however being unclear or meaningless altogether. It has a definite meaning, which can become more precise only through further elaboration and specification, including a closer definition of the context in which the concept is used. Fuzzy logic is a form of many-valued logic in which the truth values of variables may be any real number between 0 and 1. By contrast, in Boolean logic, the truth values of variables may only be the "crisp" values 0 or 1. Fuzzy logic has been employed to handle the concept of partial truth, where the truth value may range between completely true and completely false. Furthermore, when linguistic variables are used, these degrees may be managed by specific (membership) functions.

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REFERENCES

1) Klaua, D. (1965) Uber einen Ansatz zur mehrwertigen Mengenlehre. Monatsb. Deutsch. Akad. Wiss. Berlin 7, 859–876. A recent in-depth analysis of this paper has been provided by Gottwald, S. (2010). "An early approach toward graded identity and graded membership in set theory". Fuzzy Sets and Systems. 161 (18): 2369–2379.
2) Hans-Jürgen Zimmermann (2001). Fuzzy set theory—and its applications (4th ed.). Kluwer. ISBN 978-0-7923-7435-0
3) Lofti A. Zadeh, "Fuzzy Sets" Information and Control, Vol. 8, June 1965, pp 338 – 353.
4) Lotfi A Zadeh with George J. Klir and Bo Yua, Fuzzy Sets, Fuzzy Logic, and Fuzzy Systems: Selected Papers. Singapore and River Edge (N.J.): World Scientific Publishing Company, 1996.
5) Lotfi Zadeh, "Factual Information about the Impact of Fuzzy Logic". Berkeley Initiative in Soft Computing, at Electrical Engineering and Computer Sciences Department, University of Berkeley, California, circa 2014